JEE Advanced 2026

Sample Paper - 1 (Paper-2)

Time Allowed: 3 hours **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and three sub-sections as below.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

Physics

1. Two identical thin rings, each of radius R metres, are coaxially placed a distance R metres apart. If Q_1 coulomb and Q_2 coulomb, are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is: **[3]**

   a) $\frac{q(Q_1-Q_2)(\sqrt{2}-1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

   b) $\frac{q(Q_1+Q_2)(\sqrt{2}+1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

   c) Zero

   d) $\frac{q\sqrt{2}(Q_1+Q_2)}{(4\pi\varepsilon_0 R)}$

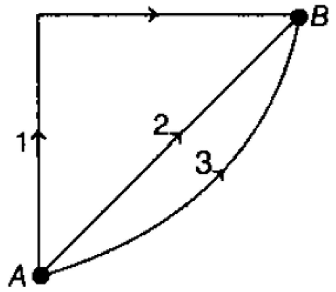
2. The ratio of the magnetic field and magnetic moment at the centre of a current-carrying circular loop is x. When both the current and radius is doubled the ratio will be: **[3]**

   a) $\frac{x}{8}$

   b) 2x

c) $\frac{x}{4}$ d) $\frac{x}{2}$

3. If W_1,W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m. Find the correct relation between W_1 , W_2 and W_3 **[3]**



a) $W_1 = W_2 = W_3$ b) $W_2 > W_1 > W_3$

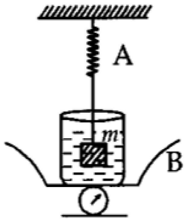
c) $W_1 > W_2 > W_3$ d) $W_1 < W_2 < W_3$

4. Which of the following sets have different dimensions? **[3]**

a) Heat, Work done, Energy b) Pressure, Young's modulus, Stress

c) Emf, Potential difference, Electric potential d) Dipole moment, Electric flux, Electric field

5. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation: **[4]**



a) the balance B will read more than 5 kg b) the balance A will read more than 2 kg

c) the balance A and B will read 2 kg and 5 kg respectively d) the balance A will read less than 2 kg and B will read more than 5 kg

6. The coordinates of a particle moving in a plane are given by x(t) = a cos (pt) and y (t) = b sin (pt) where a, b (< a) and p are positive constants of appropriate dimensions. Then **[4]**

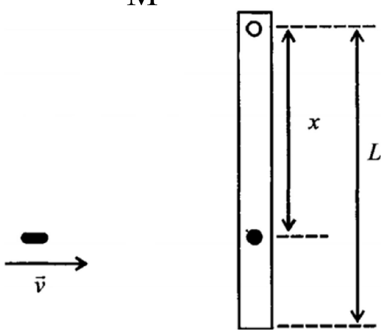
a) the acceleration of the particle is always directed towards a focus

b) the path of the particle is an ellipse

c) the velocity and acceleration of the particle are normal to each other at t$t = \frac{\pi}{(2p)}$

d) the distance travelled by the particle in time interval t = 0 to $t = \frac{\pi}{(2p)}$ is a

7. A rod of mass m and length L, pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed v strikes the rod horizontally at a distance x from its pivoted end and gets embedded in it. The combined system now rotates with angular speed ω about the pivot. The maximum angular speed ω_M is achieved for x = x_M Then **[4]**



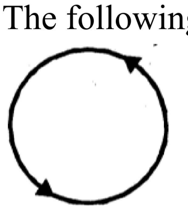
a) $\omega_M = \frac{v}{2L}\sqrt{3}$

b) $x_M = \frac{L}{\sqrt{3}}$

c) $\omega = \frac{3vx}{L^2+3x^2}$

d) $\omega = \frac{12vx}{L^2+12x^2}$

8. The following field line can never represent **[4]**



a) electrostatic field

b) induced electric field

c) magnetostatic field

d) gravitational field of a mass at rest

9. A 20 cm long string having a mass of 1.0 gm is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibration using an external vibrator of **[4]**

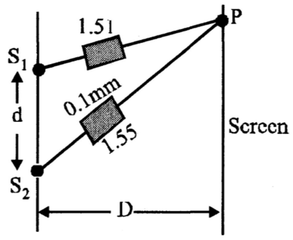
frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.

10. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 km s^{-1}, the escape speed on the surface of the planet in kms^{-1} will be **[4]**

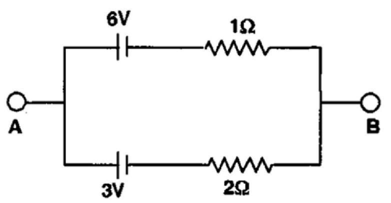
11. In a photoelectric experiment, a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$N due to the impact of the electrons. The value of n is ________. Mass of the electron $m_e = 9 \times 10^{-31}$ kg and $10eV = 1.6 \times 10^{-19}$ J. **[4]**

12. Consider an LC circuit, with inductance L = 0.1 H and capacitance $C = 10^{-3}$F, kept on a plane. The area of the circuit is $1m^2$. It is placed in a constant magnetic field of strength B_0 which is perpendicular to the plane of the circuit. At time t = 0, the magnetic field strength starts increasing linearly as $B = B_0 + \beta t$ with $\beta = 0.04$ T s^{-1}. The maximum magnitude of the current in the circuit is ________ mA. **[4]**

13. In Young's double slit experiment, two slits S_1 and S_2 are **d** distance apart and the separation from slits to screen is D (as shown in figure). Now if two transparent slabs of equal thickness 0.1 mm but refractive index 1.51 and 1.55 are introduced in the path of beam ($\lambda = 4000\overset{o}{A}$) from S_1 and S_2 respectively. The central bright fringe spot will shift by ________ number of fringes. **[4]**

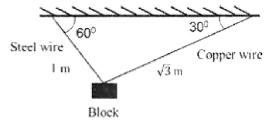


14. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is **[4]**



15. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad/sec. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, then the value of n is: **[4]**

16. A block of weight 100N is suspended by copper and steel wires of same cross-sectional area 0.5 cm^2 and, length $\sqrt{3}$m and 1m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with the ceiling are 30° and 60°, respectively. If elongation in copper wire is and elongation in steel wire is (Δl_S), then the ratio $\frac{\Delta l_c}{\Delta l_s}$ is ________. **[4]**
[Young's modulus for copper and steel are 1×10^{11} N/m^2 and 2×10^{11} N/m^2, respectively.]



Chemistry

17. The correct set of quantum numbers for the unpaired electron of chlorine atom is **[3]**

a)

n	l	m
3	1	1

b)

n	l	m
2	1	1

c)

n	l	m
2	1	0

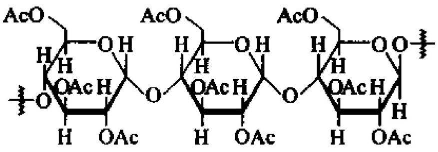
d)

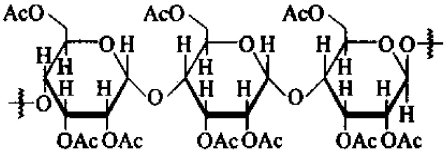
n	l	m
3	0	0

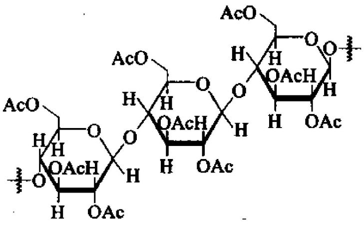
18. The rate of diffusion of methane at a given temperature is twice that of a gas X. The molecular weight of X is **[3]**

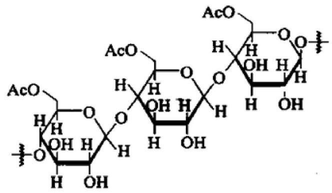
a) 4.0
b) 8.0
c) 32.0
d) 64.0

19. Cellulose upon acetylation with excess acetic anhydride/ H_2SO_4 (catalytic) gives cellulose triacetate whose structure is **[3]**

a) 

b) 

c) 

d) 

20. Calamine, malachite, magnetite and cryolite, respectively, are [3]

a) $ZnCO_3, CuCO_3, Fe_2O_3, Na_3AlF_6$

b) $ZnSO_4, Cu(OH)_2, Fe_3O_4, Na_3AlF_6$

c) $ZnSO_4, CuCO_3, Fe_2O_3, AlF_3$

d) $ZnCO_3, CuCO_3 \cdot Cu(OH)_2, Fe_3O_4, Na_3AlF_6$

21. Only two isomeric monochloro derivatives are possible for [4]

a) 2-methylpropane

b) benzene

c) 2,4-dimehylpentane

d) n-butane

22. Which of the following statement(s) is (are) true? [4]

a) Monosaccharides cannot be hydrolysed to give poly-hydroxy aldehydes and ketones.

b) Oxidation of glucose with bromine water gives glutamic acid.

c) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose.

d) The two six-membered cyclic hemiacetal forms of D- (+)-glucose are called anomers.

23. When $NaNO_3$ is heated in a closed vessel, oxygen is liberated and $NaNO_2$ is left behind. At equilibrium. [4]

a) addition of $NaNO_2$ favours reverse reaction

b) addition of $NaNO_3$ favours forward reaction

c) increasing pressure favours reverse reaction

d) increasing temperature favours forward reaction

24. The colour of the X_2 molecules of group 17 elements changes gradually from yellow to violet down the group. This is due to [4]

a) Decrease in π^* - σ^* gap down the group

b) The physical state of X_2 at room temperature changes from gas to solid down the group

c) Decrease in ionization energy down the group

d) Decrease in HOMO-LUMO gap down the group

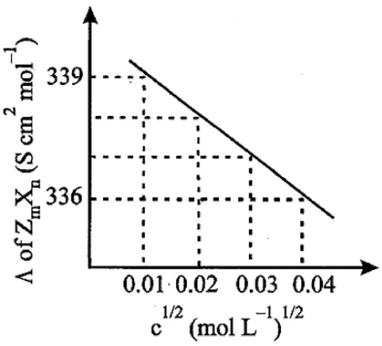
25. Consider the strong electrolytes Z_mX_n, U_mY_p and V_mX_n. Limiting molar conductivity (Λ^0) of U_mY_p and V_mX_n are 250 and 440 S cm^2 mol^{-1}, respectively. The value of (m + n + p) is ________. **[4]**

Given:

Ion	Z^{n+}	U^{p+}	V^{n+}	X^{m-}	Y^{m-}
λ^0 (S cm^2 mol^{-1})	50.0	25.0	100.0	80.0	100.0

λ^0 is the limiting molar conductivity of ions.

The plot of molar conductivity (Λ) of Z_mX_n vs $c^{\frac{1}{2}}$ is given below.



26. In the Arrhenius equation for a certain reaction, the value of A and E_a (activation energy) are 4×10^{13} sec^{-1} and 98.6 kJ mol^{-1} respectively. If the reaction is of first order, at what temperature will its half-life period be ten minutes? **[4]**

27. H_2S (5 moles) reacts completely with acidified aqueous potassium permanganate solution. In this reaction, the number of moles of water produced is x, and the number of moles of electrons involved is y. The value of (x + y) is ________. **[4]**

28. Consider the following molecules: Br_3O_8, F_2O, $H_2S_4O_6$, $H_2S_5O_6$, and C_3O_2. Count the number of atoms existing in their zero oxidation state in each molecule. Their sum is ________. **[4]**

29. To 500 cm^3 of water, 3.0×10^{-3} kg of acetic acid is added. If 23% of acetic acid is dissociated, what will be the depression in the freezing point K_f and density of water are 1.86 K kg^{-1} mol^{-1} and 0.997 g cm^{-3}, respectively. **[4]**

30. On dissolving 0.5 g of a non-volatile non-ionic solute to 39 g of benzene, its vapour pressure decreases from 650 mm Hg to 640 mm Hg. The depression of freezing point of Benzene (in K) upon addition of the solute is ________. **[4]**

(Given data: Molar mass and the molal freezing point depression constant of benzene are 78 g mol^{-1} and 5.12 K kg mol^{-1}, respectively)

31. The transformation occurring in Duma's method is given below: **[4]**

$C_2H_7N + \left(2x + \frac{y}{2}\right)CuO \rightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right)Cu$

The value of y is ________. (Integer answer)

32. Among the following, the number of elements showing only one non-zero oxidation state is: **[4]**

O, Cl, F, N, P, Sn, Tl, Na, Ti

Mathematics

33. The number of real solutions of **[3]**

$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is

a) zero
b) two
c) infinite
d) one

34. For any positive integer n, define $f_n : (0, \infty) \rightarrow \mathbb{R}$ as $f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$ for all $x \in (0, \infty)$ **[3]**

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$)

Then, which of the following statement(s) is (are) TRUE?

a) For any fixed positive integer n, $\lim_{x\to\infty} \sec^2(f_n(x)) = 1$
b) $\sum_{j=1}^{10} (1 + f'_j(0)) \sec^2(f_j(0)) = 10$
c) For any fixed positive integer n, $\lim_{x\to\infty} \tan(f_n(x)) = \frac{1}{n}$
d) $\sum_{j=1}^{5} \tan^2(f_j(0)) = 55$

35. If the system of equations x + ay = 0, az + y = 0 and ax + z = 0 has infinite solutions, then the value of a is **[3]**

a) 1
b) -1
c) 0
d) no real values

36. The function f(x) = $\frac{\log(1+ax)-\log(1-bx)}{x}$ is not defined at x = 0. The value which should be assigned to f at x = 0, so that it is continuous at x = 0, is **[3]**

a) a + b

b) log a - log b

c) a - b

d) log a + log b

37. Three lines $L_1 : \vec{r} = \lambda\hat{i}, \lambda \in R$, $L_2 : \vec{r} = \hat{k} + \mu\hat{j}, \mu \in R$ and $L_3 : \vec{r} = \hat{i} + \hat{j} + v\hat{k}, v \in R$ are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear? **[4]**

a) $\hat{k} + \hat{j}$

b) $\hat{k}$

c) $\hat{k} + \frac{1}{2}\hat{j}$

d) $\hat{k} - \frac{1}{2}\hat{j}$

38. Let a hyperbola passes through the focus of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then **[4]**

a) the equation of hyperbola is $\frac{x^2}{9} + \frac{y^2}{16} = 1$

b) vertex of hyperbola is $(5\sqrt{3}, 0)$

c) focus of hyperbola is (5, 0)

d) the equation of hyperbola is $\frac{x^2}{9} + \frac{y^2}{25} = 1$

39. A solution curve of the differential equation $(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0, x > 0$, passes through the point (1, 3). Then the solution curve **[4]**

a) does NOT intersect $y = (x + 3)^2$

b) intersects y = x + 2 exactly at two points

c) intersects y = x + 2 exactly at one point

d) intersects $y = (x + 2)^2$

40. There are three bags B_1, B_2, and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}$, $\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct? **[4]**

a) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

b) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

c) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{5}{13}$

d) Probability that the chosen ball is green equals $\frac{39}{80}$

41. Let M be a 3 × 3 matrix satisfying $M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$, $M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$, and **[4]**

$M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$

Then, the sum of the diagonal entries of M is

42. For x ∈ ℝ, then number of real roots of the equation $3x^2-4\left|x^2-1\right|+x-1=0$ is _________. **[4]**

43. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}}\times(\sqrt{7})^{\frac{1}{\log_4 7}}$ is _________. **[4]**

44. Let f be a function defined on R (the set of all real numbers) such that f' (x) = 2010 (x - 2009) (x - 2010)^2 (x - 2011)^3 (x - 2012)^4, ∀ x ∈ R. If g is a function defined on R with values in the interval (0, ∞) such that f (x) = In (g (x)), ∀ x ∈ R, then the number of points in R at which g has a local maximum is **[4]**

45. Let a and b be two non-zero real numbers. If the coefficient of x^5 in the expansion of $\left(ax^2+\frac{70}{27bx}\right)^4$ is equal to the coefficient of x^{-5} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^7$, then the value of 2b is **[4]**

46. Consider the lines L_1 and L_2 defined by **[4]**

L_1: $x\sqrt{2}+y-1=0$ and L_2 : $x\sqrt{2}-y+1=0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ_2. The line y = 2x + 1 meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the **square** of the distance between R' and S'. The value of λ^2 is _________.

47. Let f : R → R be a differentiable function such that its derivative f' is continuous and f(π) = -6. **[4]**

If F : [0, π] $\rightarrow$ R is defined by F(x) = $\int_0^x$ f(t)dt, and if $\int_0^x (f'(x) + F(x)) \cos x \, dx = 2$,

then the value of f(0) is ________.

48. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \leq p \leq 100$. For any integer n with $1 \leq n \leq 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is equal to **[4]**

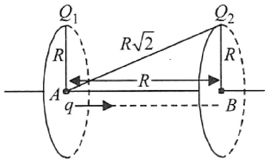
Solution

Physics

1. **(a)** $\frac{q(Q_1-Q_2)(\sqrt{2}-1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

Explanation:

The work done in moving a charge q from centre of one ring to other = change in potential energy



$W_{AB} = \mu$

$\mu_A = \left[\left(\frac{Q_1}{4\pi\varepsilon_0 R}\right) \times q + \left(\frac{Q_2}{4\pi\varepsilon_0\sqrt{R^2+R^2}}\right) q\right]$

$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}}\right]$

$\mu_B = \left[\left(\frac{Q_2}{4\pi\varepsilon_0 R}\right) q + \left(\frac{Q_1}{4\pi\varepsilon_0\sqrt{R^2+R^2}}\right) q\right]$

$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_2 + \frac{Q_1}{\sqrt{2}}\right]$

$\therefore W_{AB} = \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}} - Q_2 - \frac{Q_1}{\sqrt{2}}\right]$

$= \frac{q(Q_1-Q_2)}{4\pi\varepsilon_0 R}\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$

2. **(a)** $\frac{x}{8}$

Explanation:

The magnetic field at the centre of a circular loop of radius R carrying current I is,

$B = \frac{\mu_0 2\pi I}{4\pi R} = \frac{\mu_0 I}{2R}$

Its magnetic moment is, $M = IA = I(\pi R^2)$

$\therefore \frac{B}{M} = \frac{\mu_0 I}{2R} \times \frac{1}{I\pi R^2} = \frac{\mu_0}{2\pi R^3} = x$ (Given)

When both the current and radius is doubled, the ratio becomes

$\therefore \frac{B'}{M'} = \frac{\mu_0}{2\pi(2R)^3} = \frac{1}{8}\left(\frac{\mu_0}{2\pi R^3}\right) = \frac{x}{8}$

3. **(a)** $W_1 = W_2 = W_3$

Explanation:

Gravitational field is a conservative force field. In a conservative force field work done is path independent

$\therefore W_1 = W_2 = W_3$

4.

(d) Dipole moment, Electric flux, Electric field

Explanation:

Dipole moment = (charge) × (distance)

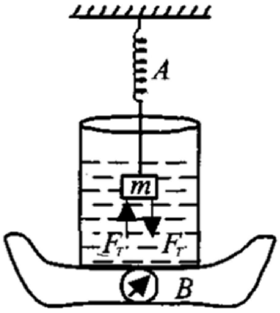
Electric flux = (electric field) × (area)

Hence, the correct option is (d).

5. **(a)** the balance B will read more than 5 kg

(d) the balance A will read less than 2 kg and B will read more than 5 kg

Explanation: When the block of mass m is inside the liquid an upthrust F_T will act on the mass which will decrease the reading on A i.e., will read less than 2 kg.



According to Newton's third law, to each and every action, there is equal and opposite reaction.

So by reaction an equal force will be exerted on the liquid of the beaker which will increase the reading in B i.e., will read more than 5 kg.

6. **(a)** the acceleration of the particle is always directed towards a focus

(b) the path of the particle is an ellipse

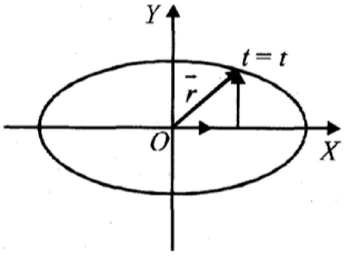
(c) the velocity and acceleration of the particle are normal to each other at t$t = \frac{\pi}{(2p)}$

Explanation: x = a cos pt ⇒ cos (pt) = $\frac{x}{a}$ ...(i)

y = b sin pt ⇒ sin (pt) = $\frac{y}{b}$ ...(ii)

Squaring and adding eqn. (i) and (ii), we get,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



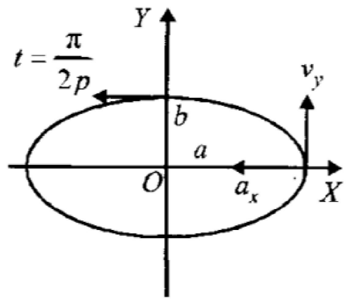
Hence path of the particle is an ellipse.

From the given equations we can find,

$\frac{dx}{dt} = v_x = -ap \sin pt$; $\frac{d^2x}{dt^2} = a_x = -ap^2 \cos pt$

$\frac{dy}{dt} = v_y = pb \cos pt$

and $\frac{d^2y}{dt^2} = a_y = -bp^2 \sin pt$



At time t = $\frac{\pi}{2p}$ or pt = $\frac{\pi}{2}$

a_x and v_y become zero (because cos $\frac{\pi}{2}$ = 0). Only v_x and a_y are left, or we can say that velocity is along negative x-axis and acceleration along negative y-axis.

Hence, at t = $\frac{\pi}{2p}$, velocity and acceleration of the particle are normal to each other.

At t = t, position of the particle

$\vec{r}(t)=x\hat{i}+y\hat{j}=a\cos pt\hat{i}+b\sin pt\hat{j}$

and acceleration of the particle

$\vec{a}(t)=a_x\hat{i}+a_y\hat{j}=-p^2[a\cos pt\hat{i}+b\sin pt\hat{j}]$

$=-p^2[x\hat{i}+y\hat{j}]=-p^2\vec{r}(t)$

Therefore, acceleration of the particle is always directed towards origin.

At t = 0, particle is at (a, 0) and at t = $\frac{\pi}{2p}$, particle is at (0, b). Therefore, the distance covered is one fourth of the elliptical path and not a.

7. **(a)** $\omega_M=\frac{v}{2L}\sqrt{3}$

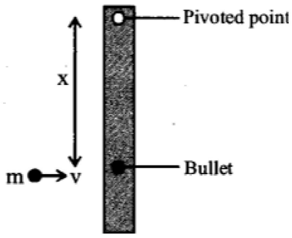
(b) $x_M=\frac{L}{\sqrt{3}}$

(c) $\omega=\frac{3vx}{L^2+3x^2}$

Explanation: From angular momentum conservation about the pivoted point.

$\mathrm{mvx}=\left(\frac{\mathrm{mL}^2}{3}+\mathrm{mx}^2\right)\omega$

[As the combined system rotates with angular speed ω about the pivot]



$\therefore\omega=\frac{mvx}{\frac{mL^2}{3}+mx^2}=\frac{3vx}{L^2+3x^2}$

$\omega=\frac{3vx}{L^2+3x^2}$

Hence option ($\omega=\frac{3vx}{L^2+3x^2}$) is correct.

For maximum angular velocity, $\frac{d\omega}{dx}=0$

$\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{\mathrm{L}^2}{\mathrm{x}}+3\mathrm{x}\right)=0$

$\Rightarrow\frac{\mathrm{L}^2}{x^2}+3=0\Rightarrow x=\frac{\mathrm{L}}{\sqrt{3}}$

$\therefore \mathrm{X_m} = \frac{\mathrm{L}}{\sqrt{3}}$

So option ($x_M = \frac{L}{\sqrt{3}}$) is correct.

$\omega_m = \frac{3vx}{L^2+3x^2} = \frac{3v\frac{L}{\sqrt{3}}}{L^2+3\left(\frac{L}{\sqrt{3}}\right)^2} = \frac{\sqrt{3}}{2L}V$

Hence option ($\omega_M = \frac{v}{2L}\sqrt{3}$) is correct.

8. **(a)** electrostatic field

   (d) gravitational field of a mass at rest

   Explanation: Only induced electric field and magnetostatic field form closed loops of field lines.

9. 5

   Explanation:

   Wave speed = $\sqrt{\frac{T}{\mu}} = \sqrt{\frac{0.5}{10^{-3}/0.2}}$ = 10 m/s

   Using, v = fλ

   $\lambda = \frac{v}{f} = \frac{10}{100}$ = 10 cm

   Distance between successive nodes = $\frac{\lambda}{2}$ = 5 cm

10. 3

    Explanation:

    $g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2}$ or $g \propto \rho R$ or $R \propto \frac{g}{\rho}$

    Now escape velocity, $v_e = \sqrt{2gR}$

    or $v_e \propto \sqrt{gR}$ or, $v_e \propto \sqrt{g \times \frac{g}{\rho}} \propto \sqrt{\frac{g^2}{\rho}}$

    $\therefore (v_e)_{planet} = (11\ \mathrm{kms^{-1}})\sqrt{\frac{6}{121} \times \frac{3}{2}} = 3\ \mathrm{kms^{-1}}$

    $\therefore$ The correct answer is 3

11. 24

    Explanation:

    Number of electrons emitted/s = $\frac{200W}{6.25\times1.6\times10^{-19}J}$

    Force, F = Rate of change of linear momentum = $N\sqrt{2mV}$

    $= \frac{200}{6.25\times1.6\times10^{-19}} \times \sqrt{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19} \times 500}$

    $\Rightarrow$ F = 24 $\times$ 10^{-4} N [$\because$ K = eV: e = 1.6 $\times$ 10^{-19} = V = 500]

    $\therefore$ n = 24

12. 4.0

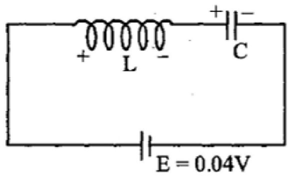
    Explanation:

    Induced Emf,

    E = $\left|\frac{d\phi}{dt}\right| = \frac{d}{dt}(BA) = \frac{Ad}{dt}[(B_0 + \beta t)]$

    = Aβ =1 $\times$ 0.04 = 0.04 volt

So, circuit can be drawn as



By KVL, $E = L\frac{di}{dt} + \frac{q}{c}$

$\Rightarrow L\frac{di}{dt} = E - \frac{q}{c}$

$\Rightarrow \frac{d^2q}{dt^2} = -\frac{1}{LC}(q - CE)$

Comparing it with equation of SHM, we get

$q = CE + A\sin(\omega t + \phi)$, where $\omega = \frac{1}{\sqrt{LC}}$

So, $i = A\,\omega\cos(\omega t + \phi)$

at $t = 0$, $q = 0$ and $i = 0$

So, $O = CE + A\sin\phi \Rightarrow A\sin\phi = -CE$ ...(i)

$O = \cos\phi \Rightarrow \phi = \frac{\pi}{2}$ ...(ii)

from (i) and (ii), we get

$A = -CE$

So, $i = -CE\,\omega\cos(\omega t + \frac{\pi}{2})$

$= CE\,\omega\sin\omega t$

Therefore, $i_{max} = CE\,\omega = 10^{-3} \times 0.04 \times \frac{1}{\sqrt{0.1\times10^{-3}}} = 4$ mA

13. 10.0

Explanation:

Path difference at P be Δx

$\Delta x = (\mu_2 - \mu_1)t = (1.55 - 1.51)0.1$ mm

$= 0.04 \times 10^{-4}$

$\Delta x = 4 \times 10^{-6} = 4\ \mu m$

Here, y is the distance of central maxima from geometric center

$y = \frac{\Delta x D}{d} = 4 \times 10^{-6}\frac{D}{d}$

fringe width $(\beta) = \frac{\lambda D}{d} = 4 \times 10^{-6}\frac{D}{d} = 4\frac{D}{d}\mu m$

$\therefore$ Central bright fringe spot will shift by 'x'

Number of shift $= \frac{y}{\beta} = \frac{4\times10^{-6}\frac{D}{d}}{4\times10^{-7}\frac{D}{d}} = 10$

14. 5.0

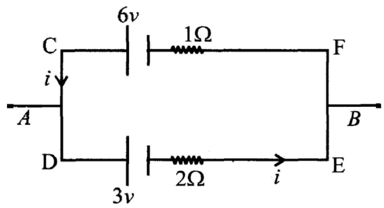
Explanation:

Let i be the current flowing in the circuit. Applying Kirchhoff's law KVL in the loop CDEFC

$-3 - 2i - i + 6 = 0 \Rightarrow 3i = 3$

$\therefore i = 1$ A

Now for upper or lower path



$V_A - 6 + 1 \times 1 = V_B$

$\therefore V_A - V_B = 5V$

or, $V_A - 3 - 2 \times 1 = V_B$

$\therefore V_A - V_B = 5\ V$

15. 4

Explanation:

Let Y is the Young's modulus of wire. In equilibrium, the tension in the wire is,

$T = mg = 0.1 \times 9.8 = 0.98$ N

Let elongation in wire in equilibrium is I_0, then,

$Y = \frac{T/A}{l_0 L} = \frac{0.98\times1}{4.9\times10^{-7}\times L_0} = \frac{2\times10^6}{l_0}$

Let us say the wire is pulled by distance x from its equilibrium position, then in this displaced position, let F is the tension in the wire, then,

$Y = \frac{F/A}{(l_0+x)/L} = \frac{FL}{A(l_0+x)}$

or $F = \frac{Y(l_0+x)A}{L}$

Restoring force on block the s, $F_{res} = F - mg$

$F_{res} = \frac{Yl_0A}{L} + \frac{Y_xA}{L} - mg$

$= \frac{YA}{L} \times x\ [\because \frac{Yl_0A}{L} = mg]$

$\therefore a = \frac{YA}{mL}x$

As $a \propto x$, so block performs SHM with time period given by:

$T = \frac{2\pi}{\sqrt{\frac{YA}{mL}}}$ and $w = \frac{2\pi}{T} = \sqrt{\frac{YA}{mL}}$

$\therefore (140)^2 = \frac{YA}{mL} = \frac{Y\times4.9\times10^{-7}}{0.1\times1}$

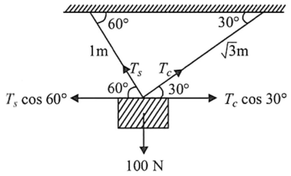
$\therefore Y = 4 \times 10^9\ N/m^2$

Hence, the required answer is, n = 4

16. 2

Explanation:

Given: $l_c = \sqrt{3}$ m; $l_s = 1$ m; $Y_c = 1 \times 10^{11}\ N/m^2$ and $T = 2 \times 10^{11}\ N/m^2$.



At equilibrium, $T_s \cos 60° = T_c \cos 30°$

$$\Rightarrow \frac{T_S}{2} = \frac{T_c\sqrt{3}}{2} \Rightarrow T_3 = \sqrt{3}T_r \Rightarrow \frac{T_c}{T_s} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{l_c}{l_s} = \frac{\sqrt{3}}{1} \text{ and } \frac{Y_c}{Y_s} = \frac{1\times10^{11}}{2\times10^{11}} = \frac{1}{2}$$

From, $Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Fl}{AY}$

Here, $A_S = A_C$

$$\therefore \frac{\Delta l_c}{\Delta l_s} = \left(\frac{T_c}{T_s}\right) \times \left(\frac{l_c}{l_s}\right) \times \left(\frac{Y_s}{Y_c}\right) = \left(\frac{1}{\sqrt{3}}\right) \times \left(\frac{\sqrt{3}}{1}\right) \times \left(\frac{2}{1}\right) = 2$$

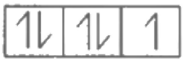
Chemistry

17. **(a)**

n	l	m
3	1	1

Explanation:

Cl (17) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^5$



The last, unpaired electron has, n = 3, l = 1(p) and m can have any of the three value (-1, 0, + 1).

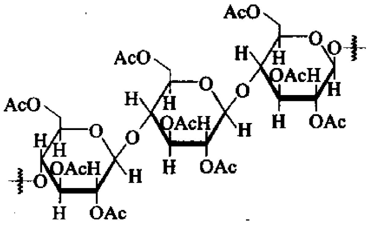
18.

(d) 64.0

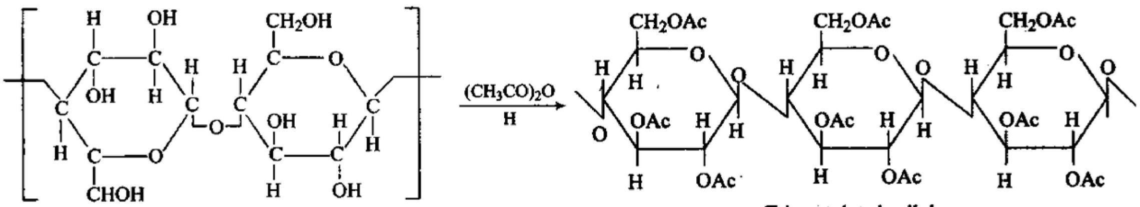
Explanation:

$$\frac{r_{CH_4}}{r_X} = 2 = \sqrt{\frac{M_X}{16}} \Rightarrow M_X = 64$$

19.

(c) 

Explanation:



Tri-acetylated cellulose

Tri-acetylated cellulose

20.

(d) $ZnCO_3, CuCO_3 \cdot Cu(OH)_2, Fe_3O_4, Na_3AlF_6$

Explanation:

$ZnCO_3, CuCO_3 \cdot Cu(OH)_2, Fe_3O_4, Na_3AlF_6$

21. **(a)** 2-methylpropane

(d) n-butane

Explanation:

a. In n-butane, Cl can add at either the first or second carbon giving two isomers.

b. $CH_3-\underset{\underset{CH_3}{|}}{C}H-CH_2-\underset{\underset{CH_3}{|}}{C}H-CH_3$ three isomers with Cl group at either of the CH_3 groups, second C-atom and third C-atom.

c. Benzene forms only one single derivative.

d. $CH_3-\underset{\underset{CH_3}{|}}{C}H-CH_3$ will again give two isomers with Cl at either one of the CH_3 groups or at the central C-atom.

22. **(a)** Monosaccharides cannot be hydrolysed to give poly-hydroxy aldehydes and ketones.

(c) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose.

(d) The two six-membered cyclic hemiacetal forms of D- (+)- glucose are called anomers.

Explanation:

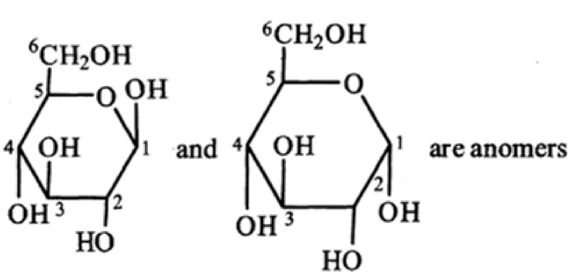
a. $\text{Glucose} \xrightarrow[H_2O]{Br_2} \text{Gluconic acid.}$

Bromine water oxidises only aldehyde group to carboxylic group. It neither oxidises -OH group nor -CO group.

b. $\underset{(+)}{\text{Sucrose}} \xrightarrow{H_2O} \underset{(+)}{\text{Glucose}} + \underset{(-)}{\text{Fructose}}$

Sucrose is dextrorotatory ($+ 66.6^{o}$) in nature but on hydrolysis it gives dextrorotatory glucose ($+52.5^{o}$) and laevorotatory fructose (-92.4^{o}). Overall solution is laevorotatory since laevorotation is more than dextrorotation.

c.



Because they differ in configuration only around C_1 position

d. Monosaccharides do not undergo further hydrolysis due to absence of glycosidic linkages.

23. **(c)** increasing pressure favours reverse reaction

(d) increasing temperature favours forward reaction

Explanation: $2NaNO_3(s) \rightleftharpoons 2NaNO_2(s) + O_2(g)$

According to Le-Chatelier principle, an increase in pressure always favours the reaction, where volume or moles decrease (i.e. reverse direction). As heat is added, i.e. reaction is endothermic and is supported in forward direction with increase in temperature. $NaNO_3$ and $NaNO_2$ both are solid. Thus, they will not effect the position of equilibrium.

24. **(a)** Decrease in π^* - σ^* gap down the group

    (d) Decrease in HOMO-LUMO gap down the group

    Explanation: Energy, E = $\frac{hc}{\lambda}$

    On moving down the group, the colour of the X_2 molecule of group 17 elements changes gradually from yellow to violet. This happens because the amount of energy required for the excitation of the halogen atom decreases down the group. HOMO (π^*) - LUMO (σ^*) gap decreases down the group that makes π^* to σ^* excitation easier. Lesser the energy gap, more is the wavelength of light absorbed and hence, lesser is the wavelength of light emitted.

25. 7

    Explanation:

    $\Lambda^\circ(U_mY_p) = m\lambda^o(U^{P+}) + p\lambda^o(Y^{m-})$

    $\Rightarrow 25m + 100p = 250$

    $\Rightarrow m + 4p = 10$ ...(1)

    $\Lambda^\circ(V_mXn) = m\lambda^o(V^{n+}) + n\lambda^o(X^{m-})$

    $\Rightarrow 100m + 80n = 440$

    $\Rightarrow 5m + 4n = 22$ ...(2)

    For electrolyte Z_mX_n from the given curve,

    $\Lambda(Z_mX_n) = \Lambda^\circ(Z_mX_n) - A\sqrt{C}$

    Slope, m = -A = $\frac{339-336}{0.01-0.04} \Rightarrow A = 100$

    For $\lambda_m = 339$ S cm^2 mol^{-1}, $\sqrt{C} = 0.01(\text{mol L} - 1)^{\frac{1}{2}}$

    $339 = \Lambda^\circ(Z_mX_n) - 100 \times 0.01$

    $\Rightarrow \Lambda^\circ(Z_mX_n) = 340$ S cm^2 mol^{-1}

    $\Rightarrow m\lambda^o(Z^{n+}) + n\lambda^o(Z^{m-}) = 340$

    $\Rightarrow 50m + 80n = 340$

    $\Rightarrow 5m + 8n = 34$ ...(3)

    From eq. (2) and (3),

    n = 3 and m = 2

    Putting value of m in eq. (1),

    p = 2

    Therefore, m + n + p = 2 + 3 + 2 = 7

26. 311.35

    Explanation:

According to Arrhenius equation

$\log k = \log A - \frac{E_a}{2.303RT}$

We know that $k = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{10\times60}$ $(t_{\frac{1}{2}} = 10 \times 60$ sec)

$= 1.555 \times 10^{-3}$

Substituting the various values in the above equation, we get

$\log 1.155 \times 10^{-3} = \log 4 \times 10^{13} - \frac{98.6}{2.303\times8.314\times10^{-3}\times T}$

On usual calculations, T = 311.35 K

27. 18

Explanation:

$2\ K\overset{+7}{Mn}O_4 + 5H_2S + 3H_2SO_4 \rightarrow K_2SO_4 + \overset{+2}{2}\ MnSO_4 + 5\ S + 8H_2O$

x = 8 (moles of H_2O produced)

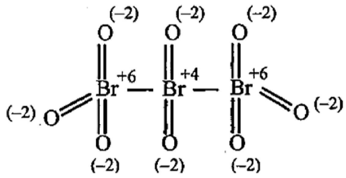
y = 14 - 4 = 10 (number of electrons involved)

x + y = 10 + 8 = 18

28. 6

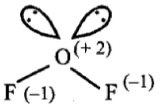
Explanation:

Br_3O_8



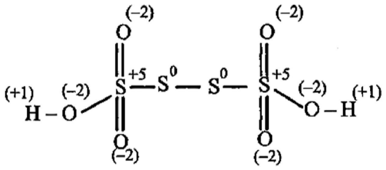
Number of atoms with zero oxidation state = 0

F_2O



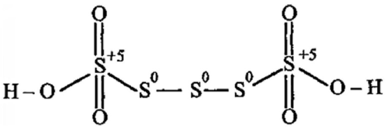
Number of atoms with zero oxidation state = 0

$H2S_4O_6$



Number of atoms with zero oxidation state = 2

$H_2S_5O_6$



Number of atoms where zero oxidation state = 3

C_3O_2

Number of atoms with zero oxidation state = 1

Sum = 2 + 3 + 1 = 6

29. 0.228

Explanation:

$\Delta T_f = i \times K_f \times m$

Weight of water = 500 $\times$ 0.997 = 498.85 g (weight = volume $\times$ density)

No. of moles of acetic acid

$= \frac{\text{Wt. of } CH_3COOH \text{ in g}}{\text{Mol. wt. of } CH_3COOH} = \frac{3 \times 10^{-3} \times 10^3}{60} = 0.05$

Since 498.5 g of water has 0.05 moles of CH_3COOH

1000 g of water has = $\frac{0.05 \times 1000}{498.5} = 0.1$

Therefore, molality of the solution = 0.1

Determination of van't Hoff factor, i

	$CH_3COOH \longrightarrow$	CH_3COO^- +	H^+
No. of moles at start	1	0	0
No. of moles at equb.	1 − 0.23	0.23	0.23

Therefore, van't Hoff factor

$= \frac{\textit{No. of particles after dissociation}}{\textit{No. of particles before dissociation}}$

$= \frac{1 - 0.23 + 0.23 + 0.23}{1} = 1.23$

Now, we know that,

$\Delta T_f = i \times K_f \times m$ = 1.23 $\times$ 1.86 $\times$ 0.1 = 0.228 K

30. 1.03

Explanation:

We know that,

$\frac{P^0 - P}{P^0} = \frac{w_2 \times m_1}{m_2 \times w_1}$

$\frac{10}{640} = \frac{0.5 \times 78}{m_2 \times 39}$

$m_2 = 64g$

$\Delta T_f = \frac{K_f \times w_2 \times 1000}{m_2 \times w_l}$

$\Delta T_f = \frac{5.12 \times 0.5 \times 1000}{64 \times 39} = 1.0256 \approx 1.03$

31. 7.0

Explanation:

$C_2H_7N + \left(2x + \frac{y}{2}\right)CuO \longrightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right)Cu$ ...(i)

$C_xH_yN_z + \left(2x + \frac{y}{2}\right)CuO \longrightarrow xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + \left(2x + \frac{y}{2}\right)Cu$ ...(ii)

Comparing (i) & (ii), we get: x = 2, y = 7, z = 1

32. 2

Explanation:

Fluorine generally shows 0 and -1 oxidation states while sodium shows 0 and +1 oxidation state.

33.

(b) two

Explanation:

Given function is

$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

Function is defined, if

i. $x(x+1)\geq 0$, since domain of square root function.

ii. $x^2+x+1\geq 0$, since domain of square root function.

iii. $\sqrt{x^2+x+1}\leq 1$, since domain of $\sin^{-1}$ function.

From (ii) and (iii), $0\leq x^2+x+1\leq 1\cap x^2+x\geq 0$

$\Rightarrow 0\leq x^2+x+1\leq 1\cap x^2+x+1\geq 1$

$\Rightarrow x^2+x+1=1$

$\Rightarrow x^2+x=0$

$\Rightarrow$ x(x + 1) = 0

$\Rightarrow$ x = 0, x = -1

34. **(a)** For any fixed positive integer n, $\lim\limits_{x\to\infty}\sec^2(f_n(x))=1$

Explanation:

$f_n(x)=\sum\limits_{j=1}^{n}\tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$

$=\sum\limits_{j=1}^{n}\tan^{-1}\left[\frac{(x+j)-(x+j-1)}{1+(x+j)(x+j-1)}\right]$

$=\sum\limits_{j=1}^{n}[\tan^{-1}(x+j)-\tan^{-1}(x+j-1)]$

$\Rightarrow f_n(x)=\tan^{-1}(x+n)-\tan^{-1}(x)$

$=\tan^{-1}\left(\frac{n}{1+x(n+x)}\right)$

$\Rightarrow f'_n(x)=\frac{1}{1+(x+n)^2}-\frac{1}{1+x^2}$

and $f_n(0)=\tan^{-1}(n)$, $\therefore \tan^2(\tan^{-1}n)=n^2$

Here x = 0 is not in the given domain, i.e., $x\in(0,\infty)$

$\lim\limits_{x\to\infty}\sec^2(f_n(x))=\lim\limits_{x\to\infty}1+\tan^2(f_n(x))$

$=1+\lim\limits_{x\to\infty}\tan^2(f_n(x))=1$

35.

(b) -1

Explanation:

Given equations x + ay = 0, az + y = 0, ax + z = 0 has infinite solutions.

$\therefore \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0$

$\Rightarrow$ 1 + a^3 = 0 or a = -1

36. **(a)** a + b

Explanation:

For f(x) to be continuous, we must have

f(0) = $\lim_{x \to 0} f(x)$

$= \lim_{x \to 0} \frac{\log(1+ax) - \log(1-bx)}{x}$

$= \lim_{x \to 0} \frac{a \log(1+ax)}{ax} + \frac{b \log(1-bx)}{-bx}$

$= a \cdot 1 + b \cdot 1 \quad \left[\text{using } \lim_{x \to 0} \frac{\log(1+x)}{x} = 1\right]$

= a + b

$\therefore$ f(0) = (a + b)

37. **(c)** $\hat{k} + \frac{1}{2}\hat{j}$

(d) $\hat{k} - \frac{1}{2}\hat{j}$

Explanation: Let any point P(λ, 0, 0) on L_1, Q(0, μ, 1) on L_2 and R(1, 1, v) on L_3

$\because$ P, Q, R are collinear, $\therefore \overrightarrow{PQ} || \overrightarrow{PR}$

$\Rightarrow \frac{\lambda}{\lambda-1} = \frac{-\mu}{-1} = \frac{-1}{-v}$

$\Rightarrow \mu = \frac{\lambda}{\lambda-1}, v = \frac{\lambda-1}{\lambda}$

Clearly from above that $\lambda \neq 0, 1$

$\therefore$ Q $\left(0, \frac{\lambda}{\lambda-1}, 1\right)$

a. For Q = $\hat{k} - \frac{1}{2}\hat{j}$

$\frac{\lambda}{\lambda-1} = -\frac{1}{2} \Rightarrow 3\lambda = +1$, which is possible.

b. For Q = $\hat{k}$

$\frac{\lambda}{\lambda-1}$ = 0 $\Rightarrow$ λ = 0, not possible

c. For Q = $\hat{k} + \hat{j}$

$\frac{\lambda}{\lambda-1}$ = 1 $\Rightarrow$ λ = λ - 1, not possible

d. For Q = $\hat{k} + \frac{1}{2}\hat{j}$

$\frac{\lambda}{\lambda-1} = \frac{1}{2}$ $\Rightarrow$ 2λ = λ - 1 $\Rightarrow$ λ = -1, which is possible.

Hence options (a) and (d) are correct and options (b) and (c) are incorrect.

38. **(a)** the equation of hyperbola is $\frac{x^2}{9} + \frac{y^2}{16} = 1$

(c) focus of hyperbola is (5, 0)

Explanation: For the given ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1 \Rightarrow$ e = $\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

$\Rightarrow$ Eccentricity of hyperbola = $\frac{5}{3}$

Let the hyperbola be $\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$ then

$B^2=A^2\left(\frac{25}{9}-1\right)=\frac{16}{9}A^2 \therefore \frac{x^2}{A^2}-\frac{9y^2}{16A^2}=1$

As it passes through focus of ellipse i.e. (3, 0)

$\therefore$ Equation of hyperbola is $\frac{x^2}{9}+\frac{y^2}{16}=1$

Its focus is (5, 0) and vertex is (3, 0).

39. **(a)** does NOT intersect $y = (x + 3)^2$

(c) intersects y = x + 2 exactly at one point

Explanation: $[(x + 2)(x + 2 + y)]\frac{dy}{dx} - y^2 = 0$, Put y = (x + 2)t

$\Rightarrow \frac{dy}{dx} = (x+2)\frac{dt}{dx} + t$

$(x + 2)^2 = 0$ or $(1 + t)\left((x+2)\frac{dt}{dx}+t\right) - t^2 = 0$

$(x + 2)(1 + t)\frac{dt}{dx} + t = 0$

$\left(\frac{1+t}{t}\right)dt=-\frac{dx}{x+2}$

ln t + t = -ln (x + 2) + c

$\Rightarrow \text{In}\left(\frac{y}{x+2}\right)+\left(\frac{y}{x+2}\right)$

$\Rightarrow$ -ln (x + 2) + c

$\ln y - \ln(x+2) + \frac{y}{x+2} = -\ln(x+2) + c$

$\text{In } y + \frac{y}{x+2} = c$

$\ln 3 + 1 = c \Rightarrow \ln y + \frac{y}{x+2} = \text{In } 3e$

i. $\text{In } y + \frac{y}{x+2} = \text{In } 3e = \ln(x+2) + 1 = \ln 3 + 1$

$\Rightarrow$ one solution

ii. $y = (x + 3)^2 \Rightarrow \ln(x+3)^2 + \frac{(x+3)^2}{x+2} = \text{In } 3 + 1$

$2\ \text{In}(x+3) + \frac{(x+2)^2+1+2(x+2)}{x+2} = \text{In } 3 + 1$

$g(x) = 2\ln(x+3) + (x+2) + 2 + \frac{1}{(x+2)} - \text{In } 3 - 1$

$g(x) = \frac{2}{(x+3)} + 1 + 0 - \frac{1}{(x+2)^2} = \frac{2(x+2)^2-(x+3)}{(x+3)(x+2)^2} + 1 > 0$

Since x > 0 given and g(0) > 0, therefore g(x) will never intersect x-axis when x > 0.

40. **(a)** Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(d) Probability that the chosen ball is green equals $\frac{39}{80}$

Explanation: $\underset{B_1}{\begin{vmatrix} R-5 \\ G-5 \end{vmatrix}}$ $\underset{B_2}{\begin{vmatrix} R-3 \\ G-5 \end{vmatrix}}$ $\underset{B_3}{\begin{vmatrix} R-5 \\ G-3 \end{vmatrix}}$

$\because P(B_1) = \frac{3}{10}, P(B_2) = \frac{3}{10}, P(B_3) = \frac{4}{10}$

$P(\frac{G}{B_1}) = \frac{5}{10}, P(\frac{G}{B_2}) = \frac{5}{8}, P\left(\frac{G}{B_3}\right) = \frac{3}{8}$

i. $P(B_3 \cap G) = P(B_3) P\left(\frac{G}{B_3}\right) = \frac{4}{10} \times \frac{3}{8} = \frac{3}{20}$

$\therefore$ Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$ is not true.

ii. $P(\frac{G}{B_3}) = \frac{3}{8}$

$\therefore$ Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$ is true.

iii. $\because P(\frac{B_3}{G})$

$= \frac{P(G/B_3)P(B_3)}{P(G/B_1)P(B_1)+P(G/B_2)P(B_2)+P(G/B_3)P(B_3)}$

$= \frac{\frac{3}{8} \times \frac{4}{10}}{\frac{5}{10} \times \frac{3}{10} + \frac{5}{8} \times \frac{3}{10} + \frac{3}{8} \times \frac{4}{10}} = \frac{\frac{12}{80}}{\frac{15}{100} + \frac{15}{80} + \frac{12}{80}}$

$= \frac{12}{80} \times \frac{400}{60+75+60} = \frac{60}{195} = \frac{4}{13}$

$\therefore$ Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{5}{13}$ is not true.

iv. $P(G) = P(\frac{G}{B_1})P(B_1) + P(\frac{G}{B_2})P(B_2) + P(\frac{G}{B_3})P(B_3)$

$= \frac{5}{10} \times \frac{3}{10} + \frac{5}{8} \times \frac{3}{10} + \frac{3}{8} \times \frac{4}{10}$

$= \frac{60+75+60}{400} = \frac{195}{400} = \frac{39}{80}$

$\therefore$ Probability that the chosen ball is green equals $\frac{39}{80}$ is true.

41. 9

Explanation:

Let $M = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$\therefore M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow M \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$M \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} a_1 - a_2 \\ b_1 - b_2 \\ c_1 - c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} a_1 + a_2 + a_3 \\ b_1 + b_2 + b_3 \\ c_1 + c_2 + c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$

$\Rightarrow a_2 = -1, b_2 = 2, c_2 = 3, a_1 - a_2 = 1, b_1 - b_2 = 1, c_1 - c_2 = -1$

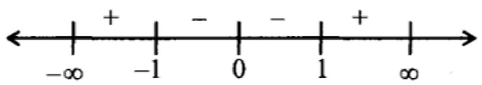
$\Rightarrow a_1 + a_2 + a_3 = 0, b_1 + b_2 + b_3 = 0, c_1 + c_2 + c_3 = 12$

$\therefore a_1 = 0, b_2 = 2, c_3 = 7$

$\Rightarrow$ Sum of diagonal elements = 0 + 2 + 7 = 9

42. 4.0

Explanation:



$3x^2 + x - 1 = 4\left|x^2 - 1\right|$

Case 1: If $x \in [-1, 1]$

$3x^2 + x - 1 = -4x^2 + 4$

$\Rightarrow 7x^2 + x - 5 = 0 \because D = 141 > 0$

$\therefore$ Equation has two roots

Case 2: If $x \in (-\infty, -1] \cup [1, \infty)$

$3x^2 + x - 1 = 4x^2 - 4$

$\Rightarrow x^2 - x - 3 = 0 \because D = 13 > 0$

$\therefore$ Equation has two roots So, total 4 roots

43. 8

Explanation:

$\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$

$= (\log_2 9)^{2\times\log_{(\log_2 9)} 2} \times 7^{\frac{1}{2}\times\log_7 4}$

$= (\log_2 9)^{\log_{(\log_2 9)} 4} \times 7^{\log_7 2} = 4 \times 2 = 8$

44. 1

Explanation:

Let $g(x) = e^{f(x)}, \forall x \in R$

$\Rightarrow g'(x) = ef(x) - f'(x)$

$\Rightarrow f'(x)$ changes its sign from positive to negative in the neighbourhood of x = 2009

$\Rightarrow f(x)$ has local maxima at x = 2009.

So, the number of local maximum is one.

45. 3.0

Explanation:

Given expansion $\left(ax^2 + \frac{70}{27b\,b}\right)^4$

$T_{r+1} = {}^4C_r\left(ax^2\right)^{4-r}\left(\frac{70}{27bx}\right)^r$

$= = {}^4C_r a^{4-r}\left(\frac{70}{27\,b}\right)^r \cdot x^{8-3r}$

Here, $8 - 3r = 5 \Rightarrow r = 1$

So, coefficient of $x^5 = {}^4C_1 a^3 \cdot \frac{70}{27\,b}$

For expansion $\left(ax - \frac{1}{bx^2}\right)^7$

$T_{r+1} = {}^7C_r\,(ax)^{7-r}\left(\frac{-1}{b^2}\right)^r = {}^7C_r\, a^{7-r}\left(\frac{-1}{b}\right)^r x^{7-3r}$

Here, 7 - 3r = -5 ⇒ r = 4

So, coefficient of $x^{-5} = {}^7C_4\, a^3\left(\frac{-1}{b}\right)^3$

A.T.Q, ${}^4C_1 a^3 \frac{70}{27b} = {}^7C_4 a^3 \cdot \frac{-1}{b^3}$

$\Rightarrow b = \frac{3}{2} \Rightarrow 2b = 3$

46. 9.0

Explanation:

Let locus point P(x, y).

∴ According to equation,

$\left|\frac{\sqrt{2}x+y-1}{\sqrt{3}}\right|\left|\frac{\sqrt{2}x-y+1}{\sqrt{3}}\right| = \lambda^2$

$\Rightarrow \left|\frac{2x^2-(y-1)^2}{3}\right| = \lambda^2$

So, C : $|2x^2 - (y - 1)2| = \lambda^2$

Let the line y = 2x + 1 meets C at two points R (x_1, y_1) and S(x_2, y_2)

⇒ $y_1 = 2x_1 + 1$ and $y_2 = 2x_2 + 1$ ...(i)

⇒ $(y_1 - y_2) = 2(x_1 - x_2)$

$\therefore RS = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$RS = \sqrt{5(x_1 - x_2)^2} = \sqrt{5}\,|x_1 - x_2|$

On solving equations curve C and line y = 2x + 1, we get

$|2x^2 - (2x)^2| = 3\lambda^2 \Rightarrow x^2 = \frac{3\lambda^2}{2}$

$\therefore RS = \sqrt{5}\left|\frac{2\sqrt{3}\lambda}{\sqrt{2}}\right| = \sqrt{30}\lambda = \sqrt{270} \Rightarrow 30\lambda^2 = 270 \Rightarrow \lambda^2 = 9$

47. 4

Explanation:

$F(x) = \int_0^x f(t)dt$

$\Rightarrow F'(x) = f(x)$

$I = \int_0^\pi f'(x)\cdot\cos x dx + \int_0^x F(x)\cos x\,dx = 2$ ...(i)

Using by parts

$\Rightarrow I = (\cos x \cdot f(x))_0^\pi + \int_0^\pi \sin x \cdot f(x)dx + \int_0^\pi F(x)\cos x dx = 2$

$\Rightarrow I = 6 - f(0) + \int_0^\pi \sin x \cdot F'(x)dx + \int_0^{\frac{0}{\pi}} F(x)\cos x dx = 2$

$I = 6 - f(0) + \int_0^{\pi} F'(x) \sin x dx + \int_0^{\pi} F(x) \cos xdx = 2$

$\Rightarrow I = 6 - f(0) + [\sin xF(x)]_0^{\pi} - \int_0^{\pi} F(x) \cos xdx + \int_0^{\pi} F(x) \cos xdx = 2$

$\Rightarrow I = 6 - f(0) + 0 = 2$

$\Rightarrow f(x) = 4$

48. 9

Explanation:

Given, $a_1 = 3$, $m = 5n$ and $a_1, a_2, \ldots,$ is an AP.

$\therefore \quad \frac{S_m}{S_n} = \frac{S_{5n}}{S_n}$ is independent of n.

$= \frac{\frac{5n}{2}[2\times3+(5n-1)d]}{\frac{n}{2}[2\times3+(n-1)d]} = \frac{5\{(6-d)+5n\}}{(6-d)+n}$

If $6 - d = 0 \Rightarrow d = 6$

$\therefore a_2 = a_1 + d = 3 + 6 = 9$

or If $d = 0$, then $\frac{S_m}{S_n}$ is independent of n.

$\therefore a_2 = 9$